Pearson Edexcel

Mark Scheme (Results)

January 2025

Pearson Edexcel International Advanced Level
In Pure Mathematics P4 (WMA14) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1	Volume = $\int_0^8 \frac{16\pi}{(x+2)^2}(\mathrm{d}x)$ oe e.g. $\pi\int_0^8 \left(\frac{4}{x+2}\right)^2 (\mathrm{d}x)$	B1
	$\int \frac{16}{(x+2)^2}\,\mathrm{d}x = \frac{-16}{(x+2)}$ or e.g. $u = x+2 \Rightarrow \int \frac{16}{(x+2)^2}\,\mathrm{d}x = \int \frac{16}{u^2}\,\mathrm{d}u = \frac{-16}{u}$	M1 A1
	$\left[\frac{-16\pi}{(x+2)}\right]_0^8 = \frac{-16\pi}{10} - \left(\frac{-16\pi}{2}\right)$ or e.g. $\left[\frac{-16\pi}{u}\right]_2^{10} = \frac{-16\pi}{10} - \left(\frac{-16\pi}{2}\right)$	M1
	$= \frac{32}{5}\pi$	A1
		(5)
		(5 marks)

B1: Correct expression for the volume in terms of x including the limits (which may be implied by later work). Condone omission of the "dx" but the correct integral with the correct limits and the 16π included must be seen or implied.

M1: For $\int \frac{\cdots}{(x+2)^2}(\mathrm{d}x) \to \frac{\cdots}{x+2}$ oe e.g. $\ldots(x+2)^{-1}$ where "…" are non-zero constants.

Or e.g. $\int \frac{\cdots}{(x+2)^2}\mathrm{d}x = \int \frac{\cdots}{u^2}(\mathrm{d}u) \to \frac{\cdots}{u}$ oe e.g. $\ldots u^{-1}$ where "…" are non-zero constants.

A1: $\int \frac{16}{(x+2)^2}(\mathrm{d}x) = \frac{-16}{(x+2)}$ or $\frac{-16}{u}$ where $u = x + 2$. Ignore any reference to π which could be 2π or e.g. 180°. The limits are not required for this mark so you can ignore any that are given.

M1: Substitutes the limits 0 and 8 into their attempted integration of $\frac{\cdots}{(x+2)^2}$ or e.g. 2 and 10 into their attempted integration of $\frac{\cdots}{u^2}$ and <u>subtracts</u> either way round. Allow this mark following poor attempts at integration, e.g. $\int \frac{\cdots}{(x+2)^2}(\mathrm{d}x) \to \ln(x+2)^2$ but it must be a "changed" function e.g. not $\int \frac{\cdots}{(x+2)^2}(\mathrm{d}x) \to \frac{\cdots}{(x+2)^2}$ and it must an expression that can be evaluated e.g. not expressions containing $\frac{1}{x}$ or e.g. $\ln x$ that cannot be evaluated at $x = 0$

If the integration is incorrect and the substitution of limits is not seen explicitly then you may need to check their answer.

A1: $\frac{32}{5}\pi$ or exact equivalent e.g. $\frac{64\pi}{10}$, 6.4π following fully correct work. $\frac{32}{5}\pi + c$ is A0.

Note $\int_0^8 \frac{16\pi}{(x+2)^2}\,\mathrm{d}x = \frac{32}{5}\pi$ on its own scores B1M0A0M0A0

Question Number	Scheme	Marks
2(a)	$2^x \to 2^x \ln 2$	B1
	$y^2 \to ...y\dfrac{dy}{dx}$	M1
	$4x^2y \to ...x^2\dfrac{dy}{dx} + ...xy$	M1
	$3 + 10y\dfrac{dy}{dx} + 4x^2\dfrac{dy}{dx} + 8xy = 10 \times 2^x \ln 2$	A1
	$\left(10y + 4x^2\right)\dfrac{dy}{dx} = 10 \times 2^x \ln 2 - 3 - 8xy \Rightarrow \dfrac{dy}{dx} = ...$	M1
	$\dfrac{dy}{dx} = \dfrac{10 \times 2^x \ln 2 - 3 - 8xy}{10y + 4x^2}$	A1
		(6)

B1: Correct differentiation $2^x \to 2^x \ln 2$

M1: For $y^2 \to ...y\dfrac{dy}{dx}$ where ... is a non-zero constant.

M1: For $4x^2y \to ...x^2\dfrac{dy}{dx} + ...xy$ where ... are non-zero constants.

A1: Correct differentiation e.g.

$3 + 10y\dfrac{dy}{dx} + 4x^2\dfrac{dy}{dx} + 8xy = 10 \times 2^x \ln 2$ or e.g. $3 + 10y\dfrac{dy}{dx} + 4x^2\dfrac{dy}{dx} + 8xy - 10 \times 2^x \ln 2 = 0$

Condone a spurious "$\dfrac{dy}{dx} =$" e.g. $\dfrac{dy}{dx} = 3 + 10y\dfrac{dy}{dx} + 4x^2\dfrac{dy}{dx} + 8xy = 10 \times 2^x \ln 2$

M1: A valid attempt to make $\dfrac{dy}{dx}$ the subject by factorising $\dfrac{dy}{dx}$ from exactly two **different** terms in $\dfrac{dy}{dx}$ which have come from differentiating $5y^2$ and $4x^2y$ and then dividing by the terms in the bracket. Note that here, 2 **different** terms means terms such as $y\dfrac{dy}{dx}$ and $x^2\dfrac{dy}{dx}$ and not e.g. $y\dfrac{dy}{dx}$ and $3y\dfrac{dy}{dx}$. Look for $(...\pm...)\dfrac{dy}{dx} = ... \Rightarrow \dfrac{dy}{dx} = ...$ which may be implied.

$\dfrac{dy}{dx}$ must be non-zero and in terms of x **and** y.

Condone slips provided the intention is clear and the above conditions are satisfied.

For those candidates who had a spurious $\dfrac{dy}{dx} = ...$ at the start, they may incorporate this in their rearrangement in which case they will have 3 terms in $\dfrac{dy}{dx}$ and so score M0.

A1: $\dfrac{dy}{dx} = \dfrac{10 \times 2^x \ln 2 - 3 - 8xy}{10y + 4x^2}$ or any equivalent correct expression e.g. $\dfrac{2^x \ln 2 - 0.3 - 0.8xy}{y + 0.4x^2}$

Note than some candidates may divide though by 10 initially e.g.

$3x + 5y^2 + 4x^2y = 10\left(2^x\right) + 35 \Rightarrow 0.3x + 0.5y^2 + 0.4x^2y = 2^x + 3.5$ and full marks are available for equivalent work in (a) and (b). Note that 2^x may appear correctly as $e^{x\ln 2}$.

2(b)	At $x = 0$ $5y^2 = 45 \Rightarrow y = 3$ $x = 0, y = 3 \Rightarrow \frac{dy}{dx} = \frac{10\times 2^0 \ln 2 - 3 - 8\times 0\times 3}{10\times 3 + 4\times 0^2} = \frac{10\ln 2 - 3}{30}$	M1 A1
		(2)

Notes:

M1: Full method of finding the gradient.

Requires:

- substituting x = 0 into the equation for C and finding a non-zero value for y. Do not be concerned about the processing as long as they are using x = 0 in the equation for C and finding a non-zero value for y
- substituting x = 0 and their non-zero y at x = 0 into their $\frac{dy}{dx}$ to find a numerical value

This may be implied by their value.

Condone slips in copying their $\frac{dy}{dx}$ from part (a) if the intention is clear.

If they have a "ln x" term on the rhs then condone "ln 0" appearing as part of their substitution but if no substitution is shown in such cases, score M0

A1: $\frac{10\ln 2 - 3}{30}$ oe e.g. $\frac{1}{3}\ln 2 - \frac{1}{10}$, $\frac{\ln 2^{10} - 3}{30}$, $\frac{\ln 1024 - 3}{30}$ **from correct work.**

Apply isw once a correct answer is seen.

There must **be no other answers** e.g. some candidates use $y = \pm 3$ to give 2 gradients.

There are many different incorrect expressions for $\frac{dy}{dx}$ that will give the correct answer here so this must follow a correct $\frac{dy}{dx}$ in part (a).

		(8 marks)

Question Number	Scheme	Marks
3(a)	$B = 6\times 4^{-\frac{1}{2}} = 3$	B1
	$3\times\left(-\frac{1}{2}\right)\left(\frac{A}{4}\right) = -\frac{1}{4} \Rightarrow A = \frac{2}{3}$	M1 A1
	$C = 3\times\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2}\left(\frac{A}{4}\right)^2 \Rightarrow C = \frac{1}{32}$	M1 A1
		(5)

Notes:

(a)

B1: For $B = 3$ which may be implied by their expansion e.g. $3\pm ...$

M1: Attempts to find A condoning the omission of their "3".

Look for $-\frac{1}{2}\times\left(\frac{Ax}{m}\right) = -\frac{1}{4}x$ o.e. where m could be 1 leading to a value for A.

Must be consistent with their $...\left(1+\frac{A}{m}x\right)^{-\frac{1}{2}}$ but condone the omission of the "3".

Note the omission of the "3" (if correct) leads to $A = 2$.

A1: $A = \frac{2}{3}$ or an exact equivalent or e.g. $0.\dot{6}$

Allow if seen embedded in $6(4+Ax)^{-\frac{1}{2}}$

M1: Attempt to find C condoning the omission of their "3".

Look for $\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\times\left(\frac{A}{m}\right)^2 = ...$ or $\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\times\left(\frac{Ax}{m}\right)^2 = ...$ oe with their numerical value of A leading to a value for C

Must be consistent with their $...\left(1+\frac{A}{m}x\right)^{-\frac{1}{2}}$ where m could be 1

Note the omission of the "3" (if correct) with a correct A leads to $C = \frac{1}{96}$.

A1: $C = \frac{1}{32}$ oe e.g. 0.03125

Allow if seen embedded in their expansion.

3(b)	$-6 < x \leq 6$	B1ft
		(1)
(c)	coefficient of x^3 is $3\times\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}\left(\frac{A}{4}\right)^3 = ...$	M1
	$-\frac{5}{1152}$	A1
		(2)

Notes:

(b)

B1ft: $-6 < x \leq 6$ but follow through on their *A* so allow for $-\frac{4}{A} < x \leq \frac{4}{A}$

<u>Condone strict or non-strict inequalities for either end</u> e.g. condone $|x| < 6$

or e.g. –6 < *x* < 6 but follow through on their *A* e.g. condone $|x| \leq \left|\frac{4}{"A"}\right|$

Accept alternative notation e.g. $x > -6$ and $x \leq 6$, $(-6, 6)$, $(-6, 6]$ etc.

(c)

M1: Attempt to find the coefficient of x^3 or the term in x^3 to obtain a value condoning the omission of their "3".

Look for an attempt at $\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)\times\left(-\frac{5}{2}\right)}{3!}\times\left(\frac{A}{m}\right)^3 = ...$ with their numerical value of *A*.

Must be consistent with their $...\left(1+\frac{A}{m}x\right)^{-\frac{1}{2}}$ from part (a) where *m* could be 1

Note the omission of the "3" with a correct *A* leads to coefficient of $-\frac{5}{3456}$.

A1: $-\frac{5}{1152}$. Condone $-\frac{5}{1152}x^3$ which may be seen embedded in an expansion.

Correct answer only with correct work in (a) scores both marks.

<u>Expansions for reference</u>

$$6(4+Ax)^{-\frac{1}{2}} = 6\times 4^{-\frac{1}{2}}\left(1+\frac{A}{4}x\right)^{-\frac{1}{2}} = 3\left(1-\frac{A}{8}x+\frac{3A^2}{128}x^2-\frac{5A^3}{1024}x^3+...\right)$$

$$= 3-\frac{3A}{8}x+\frac{9A^2}{128}x^2-\frac{15A^3}{1024}x^3+...$$

or e.g. (direct expansion)

$$6(4+Ax)^{-\frac{1}{2}} = 6\left(4^{-\frac{1}{2}}+\left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(Ax)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2}4^{-\frac{5}{2}}(Ax)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!}4^{-\frac{7}{2}}(Ax)^3+...\right)$$

$$= 6\left(\frac{1}{2}-\frac{1}{16}(Ax)+\frac{3}{256}(Ax)^2-\frac{5}{2048}(Ax)^3+...\right) = 3-\frac{3A}{8}x+\frac{9A^2}{128}x^2-\frac{15A^3}{1024}x^3+...$$

Question Number	Scheme	Marks
4(i)	$\frac{\mathrm{d}V}{\mathrm{d}t} = 70\pi$	B1
	$\frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$	B1
	$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}r} \times \frac{\mathrm{d}r}{\mathrm{d}t} \Rightarrow 70\pi = 4\pi \times 5^2 \times \frac{\mathrm{d}r}{\mathrm{d}t}$ or e.g. $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \times \frac{\mathrm{d}r}{\mathrm{d}V} \Rightarrow \frac{\mathrm{d}r}{\mathrm{d}t} = 70\pi \times \frac{1}{4\pi \times 5^2}$	M1
	$\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right) = 0.7$ (cm s^{-1}) oe e.g. $\frac{7}{10}$	A1
		(4)
	Alternative:	
	$V = 70\pi t$	B1
	$\frac{4}{3}\pi r^3 = 70\pi t$	B1
	$\frac{4}{3}\pi r^3 = 70\pi t \Rightarrow 4\pi r^2 \frac{\mathrm{d}r}{\mathrm{d}t} = 70\pi \Rightarrow 100\frac{\mathrm{d}r}{\mathrm{d}t} = 70$ or $\frac{4}{3}\pi r^3 = 70\pi t \Rightarrow r^3 = \frac{105}{2}t \Rightarrow r = \left(\frac{105}{2}t\right)^{\frac{1}{3}}$ $\left(\frac{4}{3}\pi r^3 = 70\pi t \Rightarrow \frac{500}{3}t = 70 \Rightarrow t = \frac{50}{21}\right)$ $\Rightarrow \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{3}\left(\frac{105}{2}\right)^{\frac{1}{3}} t^{-\frac{2}{3}} = \frac{1}{3}\left(\frac{105}{2}\right)^{\frac{1}{3}}\left(\frac{50}{21}\right)^{-\frac{2}{3}}$	M1
	$\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right) = 0.7$ (cm s^{-1}) oe e.g. $\frac{7}{10}$	A1

Notes:

B1: States or uses $\frac{\mathrm{d}V}{\mathrm{d}t} = 70\pi$ oe e.g. $\frac{\mathrm{d}t}{\mathrm{d}V} = \frac{1}{70\pi}$

B1: States or uses $\frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$ oe e.g. $\frac{\mathrm{d}r}{\mathrm{d}V} = \frac{1}{4\pi r^2}$ which may be seen as $\frac{\mathrm{d}V}{\mathrm{d}r} = 100\pi$ or $\frac{\mathrm{d}r}{\mathrm{d}V} = \frac{1}{100\pi}$ if $r = 5$ has been substituted.

M1: Attempts to use $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}r} \times \frac{\mathrm{d}r}{\mathrm{d}t}$ or equivalent with $r = 5$ and their $\frac{\mathrm{d}V}{\mathrm{d}r}$ and $\frac{\mathrm{d}V}{\mathrm{d}t}$ correctly placed e.g. $70\pi = 4\pi \times 5^2 \times \frac{\mathrm{d}r}{\mathrm{d}t}$ or e.g. $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}t} \times \frac{\mathrm{d}r}{\mathrm{d}V} = \frac{70\pi}{4\pi(5)^2}$

A1: $\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right) = 0.7$ oe (Units are not required but if any are given they must be correct)

(i) Alternative:

B1: States or uses $V = 70\pi t$

B1: States or uses $\frac{4}{3}\pi r^3 = 70\pi t$

M1: For either:

- Differentiating $\frac{4}{3}\pi r^3 = 70\pi t$ to obtain $...r^2\frac{\mathrm{d}r}{\mathrm{d}t} = ...$ and substituting $r = 5$ **or**
- Making r the subject from $\frac{4}{3}\pi r^3 = 70\pi t$ to obtain $r = ...t^{\frac{1}{3}}$ and then differentiating to obtain $\frac{\mathrm{d}r}{\mathrm{d}t} = ...t^{-\frac{2}{3}}$ and then substituting t obtained from $\frac{4}{3}\pi(5)^3 = 70\pi t$

A1: $\frac{\mathrm{d}r}{\mathrm{d}t} = 0.7$ oe (Units are not required but if any are given they must be correct)

(ii)	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{k}{h^3} \Rightarrow \int h^3\,\mathrm{d}h = \int k\,\mathrm{d}t \Rightarrow \frac{1}{4}h^4 = kt + c$	M1A1
	$t = 0,\ h = 4 \Rightarrow \frac{1}{4}\times 4^4 = 0 + c \Rightarrow c = 64$	M1
	$t = 5,\ h = 6 \Rightarrow \frac{1}{4}\times 6^4 = k \times 5 + 64 \Rightarrow k = (52)$	dM1
	$h = 10 \Rightarrow \frac{1}{4}(10)^4 = 52T + 64 \Rightarrow T = ...$	dddM1
	46.8 (hours) or exact e.g. $\frac{609}{13}, \frac{9744}{208}$	A1
		(6)
		(10 marks)

Notes:

M1: Separates variables and integrates both sides to obtain $...h^4 = kt(+c)$ or equivalent.
There is no need for a constant of integration for this mark.

A1: $\frac{1}{4}h^4 = kt(+c)$ or equivalent, with or without a constant of integration.

M1: Requires a constant of integration and using $t = 0,\ h = 4$ to find their c.
Condone poor integration and poor attempts to rearrange their equation for this mark.

dM1: Uses $t = 5,\ h = 6$ to find k.
It is dependent upon the **previous** mark and having two correctly placed constants.
Condone poor integration and poor attempts to rearrange their equation for this mark.

dddM1: Dependent upon **all three previous** M's.
It is for substituting $h = 10$ into their equation and finding T (or t)

A1: Awrt 46.8 or exact e.g. $\frac{609}{13}, \frac{9744}{208}$. Allow better accuracy e.g. 46.85.
It does not have to be referenced as "T" so just look for the correct value.
Units are not required but if any are given they must be correct.

See next pages for alternatives to (ii)

Alternative using definite integration:

$$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{k}{h^3}\Rightarrow\int h^3\,\mathrm{d}h=\int k\,\mathrm{d}t\Rightarrow\left[\frac{1}{4}h^4\right]_4^6=\left[kt\right]_0^5$$

$$324-64=5k\Rightarrow k=52$$

$$\left[\frac{1}{4}h^4\right]_4^{10}=\left[52t\right]_0^T\Rightarrow 2436=52T\Rightarrow T=46.8$$

Score as follows:

M1: Separates variables and integrates both sides to obtain $...h^4=kt(+c)$ or equivalent.

There is no need for a constant of integration for this mark.

A1: $\frac{1}{4}h^4=kt(+c)$ or equivalent, with or without a constant of integration.

M1: Attempts $\left[\frac{1}{4}h^4\right]_4^6=\frac{1}{4}\times 6^4-\frac{1}{4}\times 4^4=260$ Condone poor integration of h^3.

dM1: Attempts $\left[kt\right]_0^5=5k$ and solves $\left[\frac{1}{4}h^4\right]_4^6=\left[kt\right]_0^5$ to find k.

It is dependent upon the **previous** mark.

Condone poor integration and poor attempts to rearrange their equation for this mark.

dddM1: Dependent upon **all three previous** M's.

It is for attempting $\left[\frac{1}{4}h^4\right]_4^{10}=\left[52t\right]_0^T$ to find T (or t)

A1: Awrt 46.8 or exact e.g $\frac{609}{13}$, $\frac{9744}{208}$. Allow better accuracy e.g. 46.85.

It does not have to be referenced as "T" so just look for the correct value.

Units are not required but if any are given they must be correct.

Alternative when *k* is placed differently:

$$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{k}{h^3}\Rightarrow\int\frac{h^3}{k}\mathrm{d}h=\int\mathrm{d}t\Rightarrow\frac{h^4}{4k}=t+c$$

$$t=0,\ h=4\Rightarrow\frac{64}{k}=c\quad\text{or}\quad t=5,\ h=6\Rightarrow\frac{324}{k}=5+c$$

$$\frac{64}{k}=c,\ \frac{324}{k}=5+c\Rightarrow c=\frac{16}{13},\ k=52$$

$$\frac{h^4}{208}=t+\frac{16}{13}\Rightarrow\frac{10^4}{208}=t+\frac{16}{13}\Rightarrow T=\frac{609}{13}$$

Score as follows:

M1: Separates variables and integrates both sides to obtain $...\frac{h^4}{k}=t(+c)$ or equivalent.

There is no need for a constant of integration for this mark.

A1: $\frac{h^4}{4k}=t(+c)$ or equivalent, with or without a constant of integration.

M1: Requires a constant of integration and using either $t=0,\ h=4$ or $t=5,\ h=6$ to obtain an equation connecting *c* and *k*.

Condone poor integration for this mark.

dM1: Uses **both** $t=0,\ h=4$ **and** $t=5,\ h=6$ to obtain 2 equations connecting *c* and *k* and solves simultaneously to obtain a value for *c* and a value for *k*.

You do not need to be concerned how they solve their equations as long as they obtain a value for *c* and a value for *k*.

It is dependent upon the **previous** mark and having two correctly placed constants.

Condone poor integration for this mark.

dddM1: Dependent upon **all three previous** M's.

It is for substituting $h=10$ into their equation and finding *T* (or *t*)

A1: Awrt 46.8 or exact e.g $\frac{609}{13}$, $\frac{9744}{208}$. Allow better accuracy e.g. 46.85.

It does not have to be referenced as "*T*" so just look for the correct value.

Units are not required but if any are given they must be correct.

Question Number	Scheme	Marks
5(i)	$\int x^2 e^{4x} dx = \frac{1}{4}x^2 e^{4x} - \int \frac{1}{2}x\, e^{4x} dx$	M1 A1
	$= \frac{1}{4}x^2 e^{4x} - \frac{1}{8}xe^{4x} + \int \frac{1}{8} e^{4x} dx$	dM1
	$= \frac{1}{4}x^2 e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x} + c$	A1
		(4)

Notes:

(i)

M1: Attempts to integrate by parts once the correct way around. The minus must be present (not +)

Look for $ax^2 e^{4x} - \int bx\, e^{4x} dx,\quad a,\ b > 0$. Condone the omission of the "dx".

A1: $\frac{1}{4}x^2 e^{4x} - \int \frac{1}{2}x\, e^{4x} dx$ oe e.g. $\frac{1}{4}x^2 e^{4x} - \int \frac{2}{4}x\, e^{4x} dx$

dM1: Integrates $...\int x\, e^{4x} dx$ by parts the correct way around again. The second sign can be + or –

This may be seen in isolation so award for $...\int x\, e^{4x} dx = ...xe^{4x} - ...\int e^{4x} dx$

Condone the omission of the "dx".

A1: $\frac{1}{4}x^2 e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x} (+c)$ with or without a constant of integration.

Allow equivalent simplified expressions e.g. $e^{4x}\left(\frac{1}{4}x^2 - \frac{1}{8}x + \frac{1}{32}\right)(+c)$ and apply isw once a correct simplified answer is seen.

<u>Watch for "D and I" method:</u>

D	I
x^2	e^{4x}
$2x$	$\frac{1}{4}e^{4x}$
2	$\frac{1}{16}e^{4x}$
	$\frac{1}{64}e^{4x}$

In these cases score M1dM1 for obtaining:
$px^2e^{4x} \pm qxe^{4x} \pm re^{4x},\quad p,q,r > 0$
and then A1 for the **correct** first 2 terms $\frac{1}{4}x^2 e^{4x} - \frac{1}{8}xe^{4x}$
and then A1 as in the main scheme.
Note that for this approach M1A1dM0 is not possible.

(ii)	$\dfrac{2x+11}{(2x+1)(2-x)} \equiv \dfrac{A}{(2x+1)} + \dfrac{B}{(2-x)} \Rightarrow A = ..., B = ...$	M1
	$= \dfrac{4}{(2x+1)} + \dfrac{3}{(2-x)}$	A1
	$\int \dfrac{2x+11}{(2x+1)(2-x)}\,dx = 2\ln\lvert 2x+1\rvert - 3\ln\lvert 2-x\rvert$	dM1 A1ft
	$\int_4^7 \dfrac{2x+11}{(2x+1)(2-x)}\,dx = (2\ln 15 - 3\ln 5) - (2\ln 9 - 3\ln 2)$	ddM1
	$= \ln\dfrac{8}{45}$	A1
		(6)
		(10 marks)

Notes:

(ii)

M1: Attempts partial fractions. Condone slips but should be of the correct form $\dfrac{A}{(2x+1)} + \dfrac{B}{(2-x)}$

A1: Correct **fractions** $\dfrac{4}{(2x+1)} + \dfrac{3}{(2-x)}$ seen.

It is not for correct "A" and "B" unless the correct fractions are seen or implied by later work.

dM1: Integrates to a correct form e.g. $...\ln\lvert 2x+1\rvert + ...\ln\lvert 2-x\rvert$ where … are non-zero constants.

Moduli are not required, brackets will suffice but condone missing brackets for this mark e.g.

$$\int \frac{2x+11}{(2x+1)(2-x)}\,dx = 2\ln 2x+1 - 3\ln 2-x$$

A1ft: Correct integration but follow through on their A and B.

Moduli are not required, brackets will suffice <u>but must be present now</u> or implied by subsequent work e.g. when they substitute limits. Allow unsimplified e.g. $\frac{4}{2}\ln(2x+1) - 3\ln(2-x)$

ddM1: Applies the limits 4 and 7 to a function of the correct form e.g. $...\ln\lvert 2x+1\rvert + ...\ln\lvert 2-x\rvert$ or $...\ln(2x+1) + ...\ln(2-x)$ and subtracts the right way round. Brackets around the $2x + 1$ and $2 - x$ must be present or implied but condone missing brackets around the expression for the lower limit e.g. condone $\int_4^7 \dfrac{2x+11}{(2x+1)(2-x)}\,dx = 2\ln 15 - 3\ln 5 - 2\ln 9 - 3\ln 2$

and condone e.g. $\int_4^7 \dfrac{2x+11}{(2x+1)(2-x)}\,dx = 2\ln 15 - 3\ln(-5) - (2\ln 9 - 3\ln(-2))$

May be implied by e.g. $[4\ln(2x+1)]_4^7 + [3\ln(2-x)]_4^7 = 4\ln\dfrac{15}{9} + 3\ln\dfrac{5}{2}$

A1: $\ln\dfrac{8}{45}$ cao. Do **not** allow $\ln\left|-\dfrac{8}{45}\right|$.

In (ii) condone slips in copying their PF's if the work is otherwise correct e.g.

ii) $\frac{2x+11}{(2x+1)(2-x)} = \frac{A}{2x+1} + \frac{B}{2-x}$

$, \; 2x+11 = A(2-x) + B(2x+1)$

$x = 2 \Rightarrow 15 = 5B \Rightarrow B = 3$

$x = \frac{-1}{2} \Rightarrow 10 = \frac{5}{2}A \Rightarrow A = 4$

$$\frac{2x+11}{(2x+1)(2-x)} = \frac{4}{2x-1} + \frac{3}{2-x}$$

$$\int_4^7 \frac{2x+11}{(2x+1)(2-x)}\,dx = \int_4^7 \frac{4}{2x-1} + \frac{3}{2-x}\,dx$$

$$= \left[2\ln(2x-1) - 3\ln(2-x)\right]_4^7$$

$$= \left[2\ln(13) - 3\ln(-5)\right] - \left[2\ln(7) - 3\ln(-2)\right]$$

$$= 2\ln 13 - 3\ln(-5) - 2\ln 7 + 3\ln(-2)$$

$$= 2\ln\left(\frac{13}{7}\right) + 3\ln\left(\frac{2}{5}\right)$$

$$= \ln\left(\frac{13}{7}\right)^2 + \ln\left(\frac{2}{5}\right)^3$$

$$= \ln\left(\frac{1352}{6125}\right) \quad k = \frac{1352}{6125}$$

Would score M1A0M1A0M1A0 in part (ii)
If in doubt use review.

Question Number	Scheme	Marks
6	Assumption: e.g. (Assume) " n^2-4n+5 is even and n is even"	B1
	Sets e.g. $n=2k$ and attempts $(2k)^2-4(2k)+5$	M1
	$4k^2-8k+5$ (oe for their n)	A1
	Completes proof with: (i) Statement "which is odd" (ii) Reason odd E.g. $4(k^2-2k)+5$ or e.g. $4(k^2-2k+1)+1$ or e.g. $2(2k^2-4k+2)+1$ or e.g. $(2k-2)^2+1$ Condone $4k^2-8k+5=even-even+odd=odd$ (iii) A (minimal) conclusion e.g. "Hence contradiction", "so proven", "the statement is true" etc.	A1*
		(4 marks)

Notes:

Note that B0M1A1A1 is not possible.

If the same variable is used e.g. $n = 2n$ allow all but the final mark for otherwise correct work. But allow $n = 2N$ as a different variable.

B1: Sets up a suitable assumption. Must be in words.
Look for e.g. n^2-4n+5 is even and n is even or e.g. if n even then n^2-4n+5 is even.
The word "assume" is not required but there must be a statement referring to n^2-4n+5 is even and n is even.
Condone e.g. "if n^2-4n+5 is even then n is even".

M1: Uses algebra with $n = 2k$ or equivalent e.g. $n=2k+2$ or e.g. $n=4k$ and attempts n^2-4n+5

A1: $4k^2-8k+5$ or equivalent for their n such as $4(k^2-2k)+5$ or e.g. $4k^2+1$ for $n=2k+2$

A1*: Completes proof with

- a correct expression that is clearly odd
- an acceptable statement with reason
- minimal conclusion.

Note that attempts that consider a contradiction such as e.g.
(Assume) " n^2-4n+5 is odd and n is odd"
score no marks.

There will be valid approaches to the proof.
See below for some alternatives:

"n^2-4n+5 is even and n is even"

$n^2-4n+5=(n-2)^2+1$ is even so $(n-2)^2$ must be odd

If $(n-2)^2$ is odd then $(2k-2)^2$ is odd (as n is even) hence contradiction

So if n^2-4n+5 is even then n is odd

"n^2-4n+5 is even and n is even"

$n^2-4n+5=(n-2)^2+1=2k \Rightarrow (n-2)^2=2k-1$ so $(n-2)^2$ must be odd

If $(n-2)^2$ is odd then $(2a-2)^2$ is odd (as n is even) hence contradiction

So if n^2-4n+5 is even then n is odd

<u>Mark such attempts as follows:</u>

B1: As main scheme

M1: Completes the square on n^2-4n+5 to obtain $(n-2)^2+\ldots$ or uses $n^2-4n+5=n(n-4)+\ldots$

A1: $n^2-4n+5=(n-2)^2+1$ or e.g. $n^2-4n+5=n(n-4)+5$

A1: Fully reasoned and convincing argument with a (minimal) conclusion

If you are unsure if a particular response deserves merit then use review.

Question Number	Scheme	Marks
7	$x = 4\sin\theta \Rightarrow \dfrac{dx}{d\theta} = 4\cos\theta$	B1
	$\displaystyle\int \frac{1}{(16-x^2)^{\frac{3}{2}}}\,dx = \int \frac{4\cos\theta}{(16-16\sin^2\theta)^{\frac{3}{2}}}\,d\theta = \int \frac{4\cos\theta}{64\cos^3\theta}\,d\theta$	M1 A1
	$= \dfrac{1}{16}\tan\theta$	dM1
	Uses limits of $\frac{\pi}{6}$ and $\frac{\pi}{3}$ or 30° and 60° within their attempted integration and subtracts the right way round e.g. $f\left(\frac{\pi}{3}\right) - f\left(\frac{\pi}{6}\right)$ or $f(60°) - f(30°)$	M1
	$= \dfrac{1}{16}\tan\dfrac{\pi}{3} - \dfrac{1}{16}\tan\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{24}$	A1
		(6 marks)

Notes:

B1: States or uses $\dfrac{dx}{d\theta} = 4\cos\theta$ o.e. e.g. $dx = 4\cos\theta d\theta$. This may be seen within the integrand.

M1: Simplifies $(16-x^2)^{\frac{3}{2}}$ using the given substitution to $k\cos^3\theta$ using the Pythagorean identity

This may be implied if the work is correct and they proceed correctly to e.g. $\dfrac{1}{16}\displaystyle\int \cos^{-2}\theta\,d\theta$

A1: $\displaystyle\int \frac{1}{(16-x^2)^{\frac{3}{2}}}\,dx = \int \frac{4\cos\theta}{64\cos^3\theta}\,d\theta$ or exact equivalent e.g. $\dfrac{1}{16}\displaystyle\int \cos^{-2}\theta\,d\theta$ or $\displaystyle\int \frac{1}{(4\cos\theta)^2}\,d\theta$

or e.g. $\displaystyle\int \frac{4\cos\theta}{(4\cos\theta)(4\cos\theta)(4\cos\theta)}\,d\theta$

Condone if the "$d\theta$" is missing but is otherwise correct.

dM1: For $\displaystyle\int k\frac{\cos\theta}{\cos^3\theta}\,d\theta = \ldots\tan\theta$

M1: Uses limits of $\frac{\pi}{6}(30°)$ and $\frac{\pi}{3}(60°)$ correctly with their attempted integration.

Condone poor integration here.

A1: $\dfrac{\sqrt{3}}{24}$ or exact equivalent e.g. $\dfrac{1}{8\sqrt{3}}$

Question Number	Scheme	Marks
8(a)	States or implies that $a = -3$ **or** $b = 10$	B1
	States or implies that $a = -3$ **and** $b = 10$	B1
		(2)
(b)	Attempts $= \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = ...$ either way around	M1
	$\overrightarrow{AB} = \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix}$ or $12\mathbf{i} + 6\,\mathbf{j} - 3\mathbf{k}$	A1
		(2)
(c)	Attempts $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	Attempts $\overrightarrow{AC}.\overrightarrow{AB} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix} \bullet \begin{pmatrix} 12 \\ 6 \\ -3 \end{pmatrix} = 72 + 60 + 18$	dM1
	Attempts $\overrightarrow{AC}.\overrightarrow{AB} = \lvert AC \rvert \lvert AB \rvert \cos\theta \Rightarrow 150 = \sqrt{172} \times \sqrt{189} \cos\theta \Rightarrow \theta = ...$ $\left(NB \quad \sqrt{172} = 2\sqrt{43}, \ \sqrt{189} = 3\sqrt{21}\right)$	ddM1
	$\theta =$ awrt $33.7°$	A1
	(c) Alternative using the cosine rule:	
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$	M1
	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ "-3" \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ -6 \end{pmatrix}$ **and** $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix} - \begin{pmatrix} "10" \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \\ -3 \end{pmatrix}$ $\Rightarrow AB^2 = 12^2 + 6^2 + 3^2,\ BC^2 = 6^2 + 4^2 + 3^2,\ AC^2 = 6^2 + 10^2 + 6^2$	dM1
	$BC^2 = AB^2 + AC^2 - 2AB \times AC\cos\theta$ $61 = 189 + 172 - 2\sqrt{189}\sqrt{172}\cos\theta \Rightarrow \cos\theta = ...$	ddM1
	$\theta =$ awrt $33.7°$	A1
		(4)

(a)

B1: States or implies that $a = -3$ **or** $b = 10$ (Note that a comes from $\lambda = -1$ and b from $\lambda = 2$)

B1: States or implies that $a = -3$ **and** $b = 10$ (Note that a comes from $\lambda = -1$ and b from $\lambda = 2$)

In the rest of the question, you can condone slips in writing down vectors as long as the intention is clear.

(b)

M1: Attempts to subtract their vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ either way round.
If no method is shown, it can be implied by at least 2 correct components for their vectors.

A1: Correct vector using correct notation.

Do **not** allow as coordinates and do **not** allow $\begin{pmatrix} 12\mathbf{i} \\ 6\mathbf{j} \\ -3\mathbf{k} \end{pmatrix}$ but apply isw once a correct <u>vector</u> is seen.

(c)

M1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around

dM1: Attempts to find the scalar product of $\pm\overrightarrow{AC}$ and their vector $\pm\overrightarrow{AB}$
This may be implied by their value so you may need to check.
If the value is incorrect and no method is shown score M0

ddM1: Full attempt to find angle *CAB* using the scalar product of $\pm\overrightarrow{AC}$ and their vector $\pm\overrightarrow{AB}$

A1: $\theta =$ awrt 33.7° and no other angles (Degrees symbol **not** required)

Note that in (a) they can use any multiples of $\pm\overrightarrow{AC}$ and $\pm\overrightarrow{AB}$ to find the required angle.

Alternative:

M1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around

dM1: Attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OA}$ either way around **and** attempts to subtract $\overrightarrow{OC}$ and their vector $\overrightarrow{OB}$ either way around and attempts the lengths or lengths^2 of *AB*, *BC* and *AC*

ddM1: Full attempt to find angle *CAB* using the cosine rule

A1: $\theta =$ awrt 33.7° and no other angles (Degrees symbol **not** required)

(d)	Attempts a correct method of finding one position for *D*. See notes for possible approaches.	M1
	$(22, 9, -2)$ **or** $(-26, -15, 10)$	A1
	Attempts a correct method of finding both positions for *D*. See notes for possible approaches.	dM1
	$(22, 9, -2)$ **and** $(-26, -15, 10)$	A1
		(4)
		(12 marks)

(d)

M1: Attempts a complete and correct method for finding one position for *D*.
(May be implied by at least 2 correct or correct ft components)

<u>Examples:</u>

Starting from $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$: $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 5\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ **or** $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 7\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

Starting from *A* $\begin{pmatrix} -2 \\ \text{"}-3\text{"} \\ 4 \end{pmatrix}$: $\begin{pmatrix} -2 \\ \text{"}-3\text{"} \\ 4 \end{pmatrix} + 6\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ **or** $\begin{pmatrix} -2 \\ \text{"}-3\text{"} \\ 4 \end{pmatrix} - 6\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

Starting from *B* $\begin{pmatrix} \text{"}10\text{"} \\ 3 \\ 1 \end{pmatrix}$: $\begin{pmatrix} \text{"}10\text{"} \\ 3 \\ 1 \end{pmatrix} + 3\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ **or** $\begin{pmatrix} \text{"}10\text{"} \\ 3 \\ 1 \end{pmatrix} - 9\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

Note that some candidates may use their $\overrightarrow{AB}$ rather than the direction vector e.g.

Starting from $\begin{pmatrix}2\\-1\\3\end{pmatrix}$: $\begin{pmatrix}2\\-1\\3\end{pmatrix}+\frac{5}{3}\overrightarrow{AB}$ **or** $\begin{pmatrix}2\\-1\\3\end{pmatrix}-\frac{7}{3}\overrightarrow{AB}$

Starting from A $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}$: $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}+2\overrightarrow{AB}$ **or** $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}-2\overrightarrow{AB}$

Starting from B $\begin{pmatrix}"10"\\3\\1\end{pmatrix}$: $\begin{pmatrix}"10"\\3\\1\end{pmatrix}+\overrightarrow{AB}$ **or** $\begin{pmatrix}"10"\\3\\1\end{pmatrix}-3\overrightarrow{AB}$

A1: One correct point $(22, 9, -2)$ or $(-26, -15, 10)$

Condone if given as a vector e.g. $22\mathbf{i}+9\mathbf{j}-2\mathbf{k}$ or $-26\mathbf{i}-15\mathbf{j}+10\mathbf{k}$

dM1: Attempts a complete and correct method for finding both possible positions for D.

(May be implied by at least 2 correct or correct ft components)

Examples:

Starting from $\begin{pmatrix}2\\-1\\3\end{pmatrix}$: $\begin{pmatrix}2\\-1\\3\end{pmatrix}+5\begin{pmatrix}4\\2\\-1\end{pmatrix}$ **and** $\begin{pmatrix}2\\-1\\3\end{pmatrix}-7\begin{pmatrix}4\\2\\-1\end{pmatrix}$

Starting from A $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}$: $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}+6\begin{pmatrix}4\\2\\-1\end{pmatrix}$ **and** $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}-6\begin{pmatrix}4\\2\\-1\end{pmatrix}$

Starting from B $\begin{pmatrix}"10"\\3\\1\end{pmatrix}$: $\begin{pmatrix}"10"\\3\\1\end{pmatrix}+3\begin{pmatrix}4\\2\\-1\end{pmatrix}$ **and** $\begin{pmatrix}"10"\\3\\1\end{pmatrix}-9\begin{pmatrix}4\\2\\-1\end{pmatrix}$

Note that some candidates may use their $\overrightarrow{AB}$ rather than the direction vector e.g.

Starting from $\begin{pmatrix}2\\-1\\3\end{pmatrix}$: $\begin{pmatrix}2\\-1\\3\end{pmatrix}+\frac{5}{3}\overrightarrow{AB}$ **and** $\begin{pmatrix}2\\-1\\3\end{pmatrix}-\frac{7}{3}\overrightarrow{AB}$

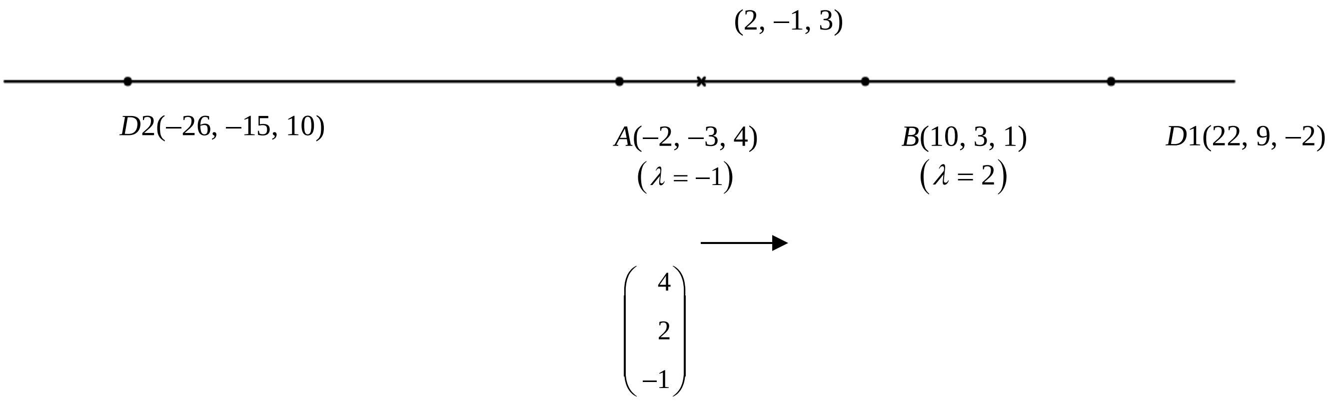
Starting from A $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}$: $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}+2\overrightarrow{AB}$ **and** $\begin{pmatrix}-2\\"-3"\\4\end{pmatrix}-2\overrightarrow{AB}$

Starting from B $\begin{pmatrix}"10"\\3\\1\end{pmatrix}$: $\begin{pmatrix}"10"\\3\\1\end{pmatrix}+\overrightarrow{AB}$ **and** $\begin{pmatrix}"10"\\3\\1\end{pmatrix}-3\overrightarrow{AB}$

A1: Gives both possible coordinates $(-26, -15, 10)$ and $(22, 9, -2)$

Condone if given as vectors e.g. $22\mathbf{i}+9\mathbf{j}-2\mathbf{k}$ and $-26\mathbf{i}-15\mathbf{j}+10\mathbf{k}$

Configuration in (d):



Note that there may be more convoluted methods in (d) e.g.

$$\text{Area } CAD = 2 \times \text{Area } CAB \Rightarrow AD = 2\sqrt{12^2+6^2+3^2} \Rightarrow AD^2 = 4\times 189$$

$$\overrightarrow{AD} = \begin{pmatrix}2\\-1\\3\end{pmatrix} + \lambda\begin{pmatrix}4\\2\\-1\end{pmatrix} - \begin{pmatrix}-2\\-3\\4\end{pmatrix} = \begin{pmatrix}4\lambda+4\\2\lambda+2\\-\lambda-1\end{pmatrix}$$

$$(4\lambda+4)^2+(2\lambda+2)^2+(-\lambda-1)^2 = 756$$

$$\Rightarrow 21\lambda^2+42\lambda-735=0 \Rightarrow \lambda = 5,\ -7$$

$$\overrightarrow{OD} = \begin{pmatrix}2\\-1\\3\end{pmatrix} + 5\begin{pmatrix}4\\2\\-1\end{pmatrix} \quad \textbf{or} \quad \begin{pmatrix}2\\-1\\3\end{pmatrix} - 7\begin{pmatrix}4\\2\\-1\end{pmatrix}$$

$$\rightarrow (-26, -15, 10) \text{ and } (22, 9, -2)$$

In such cases, marks can be awarded as above for:

- **M1: A complete and correct method to find one position for *D***
  (May be implied by at least 2 correct or correct ft components)
- **A1: One correct position for *D***
- **dM1: A complete and correct method to find both positions for *D***
  (May be implied by at least 2 correct or correct ft components)
- **A1: As main scheme**

If you are in any doubt whether a method is sound or not use review.

<u>Can also be done via the area e.g.</u>

$$\text{Area } CAB = \frac{1}{2}AB \times AC \sin\theta = \frac{1}{2}\sqrt{189}\sqrt{172}\sin\theta$$

$$\cos\theta = \frac{25}{\sqrt{43}\sqrt{21}} \Rightarrow \sin\theta = \sqrt{\frac{278}{903}}$$

$$\text{Area } CAD = \frac{1}{2}AD \times AC \sin\theta = \sqrt{189}\sqrt{172}\sqrt{\frac{278}{903}}$$

$$\overrightarrow{AD} = \begin{pmatrix} 4\lambda+4 \\ 2\lambda+2 \\ -\lambda-1 \end{pmatrix} \therefore \frac{1}{2}\sqrt{(4\lambda+4)^2+(2\lambda+2)^2+(-\lambda-1)^2}\sqrt{172}\sqrt{\frac{278}{903}} = \sqrt{189}\sqrt{172}\sqrt{\frac{278}{903}}$$

$$\Rightarrow (4\lambda+4)^2+(2\lambda+2)^2+(-\lambda-1)^2 = 756$$

$$\Rightarrow 21\lambda^2+42\lambda-735=0 \Rightarrow \lambda = 5, -7$$

$$\overrightarrow{OD} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + 5\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} \quad \textbf{or} \quad \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - 7\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

$$\rightarrow (-26, -15, 10) \text{ and } (22, 9, -2)$$

The same marking principles apply

- **M1: A complete and correct method to find one position for *D***
  (May be implied by at least 2 correct or correct ft components)
- **A1: One correct position for *D***
- **dM1: A complete and correct method to find both positions for *D***
  (May be implied by at least 2 correct or correct ft components)
- **A1: As main scheme**

For this method it is unlikely that candidates will work in exact terms and will revert to decimals.
This is acceptable but is unlikely to result in correct exact coordinates but the method marks are available as long as the method is complete and correct.

If candidates do work in decimals and then round their inexact values for the coordinates to the correct ones the A marks should be withheld.

Question Number	Scheme	Marks
9(a)(i)	$\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}=\dfrac{3\sin^2 t\cos t}{-4\sin 2t}$	M1A1
	$\dfrac{3\sin^2 t\cos t}{-4\sin 2t}=\dfrac{3\sin^2 t\cos t}{-8\sin t\cos t}=-\dfrac{3}{8}\sin t$ or e.g. $\dfrac{3\sin^2 t\cos t}{-4\sin 2t}=\dfrac{\frac{3}{2}\sin 2t\sin t}{-4\sin 2t}=-\dfrac{3}{8}\sin t$	A1
(ii)	$t=\dfrac{\pi}{6}\quad\Rightarrow\quad x=1,\quad y=\dfrac{1}{8}$	B1
	$m=-\dfrac{3}{8}\sin\left(\dfrac{\pi}{6}\right)=-\dfrac{3}{16}\ \Rightarrow y-\dfrac{1}{8}=-\dfrac{3}{16}(x-1)$	M1
	$3x+16y-5=0$ *	A1*
		(6)
(b)	$3(2\cos 2t)+16(\sin^3 t)-5=0$	M1
	$6(1-2\sin^2 t)+16\sin^3 t-5=0$	dM1
	$16\sin^3 t-12\sin^2 t+1=0$	A1
	$(2\sin t-1)^2(4\sin t+1)=0\Rightarrow \sin t=-\dfrac{1}{4}$	ddM1
	$\sin t=-\dfrac{1}{4}\Rightarrow x=2\cos 2t=..$ **and** $y=\sin^3 t=...$	dddM1
	$Q\left(\dfrac{7}{4},-\dfrac{1}{64}\right)$	A1
		(6)
		(12 marks)

Notes:

(a)(i) **Note that (a)(i) is now being marked as M1A1A1 not M1dM1A1**

M1: Attempts to use the rule $\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}$. Condone incorrect attempts on $\dfrac{dy}{dt}$ and $\dfrac{dx}{dt}$

Note that x and/or y may have been adapted before differentiation.

A1: Correct $\dfrac{dy}{dx}$ in any form **from correct work.**

A1: Achieves $\dfrac{dy}{dx}=-\dfrac{3}{8}\sin t$ **<u>from fully correct work so must follow M1A1 so M1A0A1 is not possible.</u>**

You may need to check the working carefully as the value of k can be deduced from the given tangent equation.

e.g. $3x+16y-5=0\Rightarrow m=-\dfrac{3}{16},\ m=k\sin\dfrac{\pi}{6}\Rightarrow k=-\dfrac{3}{8}$

(a)(ii)

B1: Correct coordinates for P $x=1,\ y=\frac{1}{8}$

M1: Correct method for tangent. It is awarded for:

- substituting $t=\frac{\pi}{6}$ into their $k\sin t$ or their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to find m
- using $y-y=m\left(x-x_1\right)$ where x_1 and y_1 are their values of x and y when $t=\frac{\pi}{6}$ or using $y=mx+c$ with their values of x and y when $t=\frac{\pi}{6}$ and proceeds to $c=$ ...

A1*: cso. Proceeds with a correct method and correct work to the given answer with sufficient working. Note that this cso so that if $-\frac{3}{8}\sin t$ has been obtained fortuitously or from incorrect work in part (i), this mark should be withheld.

(b)

M1: Substitutes $x = 2\cos 2t$ and $y = \sin^3 t$ into $3x+16y-5=0$
Condone slips as long as the intention is clear.

dM1: Uses $\cos 2t=\pm 1\pm 2\sin^2 t$ or equivalent work to obtain a cubic equation in $\sin t$.
The equivalent work could be e.g. $6\cos 2t=6\left(\cos^2 t-\sin^2 t\right)=6\left(1-\sin^2 t-\sin^2 t\right)$ etc.

A1: Correct cubic equation with terms collected and all on one side e.g. $16\sin^3 t-12\sin^2 t+1=0$
The "= 0" may be implied by later work e.g. by their attempt to solve.

ddM1: Solves cubic equation via any appropriate means including via a calculator to obtain $\sin t=\alpha$ where $|\alpha|<1$ and $\alpha\neq\frac{1}{2}$

dddM1: Substitutes their $\sin t=-\frac{1}{4}$ into both $x = 2\cos 2t$ and $y = \sin^3 t$ to obtain a value for x and y

This may be using appropriate identities for finding x or via a calculator e.g. $x=2\cos\left(2\sin^{-1}\left(-\frac{1}{4}\right)\right)$

A1: $Q\left(\frac{7}{4},-\frac{1}{64}\right)$ Allow any equivalent exact values and allow as $x = \ldots,\ y = \ldots$

Note that it is possible to answer (b) by eliminating *t*:

1. Via *x*:

$$x = 2\cos 2t = 2\left(1-2\sin^2 t\right) \Rightarrow \sin^2 t = \frac{2-x}{4} \Rightarrow y = \sin^3 t = \left(\frac{2-x}{4}\right)^{\frac{3}{2}}$$

$$3x + 16\left(\frac{2-x}{4}\right)^{\frac{3}{2}} - 5 = 0 \Rightarrow 16\left(\frac{2-x}{4}\right)^{\frac{3}{2}} = 5-3x \Rightarrow 4(2-x)^3 = (5-3x)^2$$

$$\Rightarrow 4x^3 - 15x^2 + 18x - 7 = 0$$

$$\Rightarrow x = \frac{7}{4} \Rightarrow y = -\frac{1}{64}$$

Score as:

M1: Uses the x coordinate and $\cos 2t = \pm 1 \pm 2\sin^2 t$ or equivalent work to find an equation connecting

$\sin t$ and x e.g. $\sin^2 t = \frac{2-x}{4}$ and uses this to find y in terms of x.

dM1: Substitutes their y in terms of x into $3x + 16y - 5 = 0$ to obtain an equation in terms of x only.

A1: Correct cubic equation with terms collected and all on one side e.g. $4x^3 - 15x^2 + 18x - 7 = 0$

The "= 0" may be implied by later work e.g. by their attempt to solve.

ddM1: Solves cubic equation via any appropriate means including via a calculator to obtain a value for x where $x \neq 1$

dddM1: Uses their value of x in $3x + 16y - 5 = 0$ to find a value of y.

A1: $Q\left(\frac{7}{4}, -\frac{1}{64}\right)$ Allow any equivalent exact values and allow as $x = \ldots$, $y = \ldots$

2. Via *y*:

$$y = \sin^3 t \Rightarrow \sin t = y^{\frac{1}{3}},\; x = 2\left(1-2\sin^2 t\right) = 2 - 4y^{\frac{2}{3}}$$

$$3\left(2-4y^{\frac{2}{3}}\right) + 16y - 5 = 0 \Rightarrow 12y^{\frac{2}{3}} = 16y + 1 \Rightarrow 1728y^2 = (16y+1)^3$$

$$\Rightarrow 4096y^3 - 960y^2 + 48y + 1 = 0$$

$$\Rightarrow y = -\frac{1}{64} \Rightarrow x = \frac{7}{4}$$

Score as:

M1: Uses the y coordinate to find an equation connecting $\sin t$ and y e.g. $\sin t = y^{\frac{1}{3}}$ and uses this to find x in terms of y.

dM1: Substitutes their x in terms of y into $3x + 16y - 5 = 0$ to obtain an equation in terms of y only.

A1: Correct cubic equation with terms collected and all on one side e.g. $4096y^3 - 960y^2 + 48y + 1 = 0$

The "= 0" may be implied by later work e.g. by their attempt to solve.

ddM1: Solves cubic equation via any appropriate means including via a calculator to obtain a value for *y* where $y \neq \frac{1}{8}$

dddM1: Uses their value of *y* in $3x+16y-5=0$ to find a value of *x*.

A1: $Q\left(\frac{7}{4}, -\frac{1}{64}\right)$ Allow any equivalent exact values and allow as *x* = …, *y* = …

Note that some candidates are bypassing the cubic and clearly using the "solve" function on their calculator. This is not satisfying the rubric of the question or the rubric on the front of the question paper. These will be marked as special cases.

Examples:

$$x = 2\cos 2t = 2\left(1-2\sin^2 t\right) \Rightarrow \sin^2 t = \frac{2-x}{4} \Rightarrow y = \sin^3 t = \left(\frac{2-x}{4}\right)^{\frac{3}{2}}$$

$$3x+16\left(\frac{2-x}{4}\right)^{\frac{3}{2}}-5=0 \Rightarrow x = \frac{7}{4}$$

$$3\left(\frac{7}{4}\right)+16y-5=0 \Rightarrow y = -\frac{1}{64}$$

Scores SC M1M1A0M0M1A1

$$x = 2\cos 2t \Rightarrow t = \frac{1}{2}\arccos\frac{x}{2} \Rightarrow y = \sin^3\left(\frac{1}{2}\arccos\frac{x}{2}\right)$$

$$3x+16\sin^3\left(\frac{1}{2}\arccos\frac{x}{2}\right)-5=0 \Rightarrow x = \frac{7}{4}$$

$$3\left(\frac{7}{4}\right)+16y-5=0 \Rightarrow y = -\frac{1}{64}$$

Scores SC M1M0A0M0M1A1

In such cases award marks as follows:

M1: Uses the *x* coordinate and $\cos 2t = \pm 1 \pm 2\sin^2 t$ or equivalent work to find an equation connecting $\sin t$ and *x* e.g. $\sin^2 t = \frac{2-x}{4}$ and uses this to find *y* in terms of *x*.

dM1: Substitutes their *y* in terms of *x* into $3x+16y-5=0$ to obtain an equation in terms of *x* only that is algebraic i.e. does not involve inverse trig functions.

A0: Not available

M0: Not available

M1: Uses their value of *x* in $3x+16y-5=0$ to find a value of *y*.

A1: $Q\left(\frac{7}{4}, -\frac{1}{64}\right)$ Allow any equivalent exact values and allow as *x* = …, *y* = …